# Ya Selection in Kyudo

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**I. Common sense selection of ya and why it matters**

The traditional selection of ya has a certain common sense as far as it goes: For "good" ya performance, the ya stiffness should increase as the strength of the yumi and length of ya increase. What are the markers of "good" ya performance? Bending oscillations of the flying ya induced by Hanare should rapidly attenuate so we see a "clean" ya flight. Beyond this, there is an *objective* requirement specifically related to Kyudo. On page 71 of the Kyohan it says: "In the full draw (Kai) the arrow must be directed along a line exactly to the center of the target." Due to the gravity-induced downward arc of the ya trajectory, this should be amended somewhat to say: "The ya at Kai should lie in a vertical plane which bisects the target." The technique of Nerai (aiming) which produces this alignment is a common challenge to many Kyudoka. For instance, a Kyudoka who hits using a consistent out of plane alignment. The implicit assumption is that correct alignment at Kai followed by well executed Hanare results in "correct hitting." This outcome may require the appropriate stiffness of ya relative to yumi strength and ya length. What happens if the ya's flexibility is too soft? It is not just exaggerated bending oscillations of the flying arrow. Conventional wisdom says that the ya lands to the *left* of the aiming point. In traditional Kyudo language, "the ya lands behind the mato."

Is that so? We perform shooting tests. The first step is to develop a sight picture corresponding to correct alignment at Kai. For the "half moon" sight picture illustrated on p. 71 of the Kyohan, it is clear that the ya itself is *not* part of it. This is similar to Olympic archery: The dominant eye sees the target framed by the scope and the arrow not at all. The illustration in the Kyohan suggests that the rattan above the grip can be a de-facto ruler to set the elevation, so it is analogous to the scope in Olympic archery. Here, the shooting test borrows a technique from western bare bow form. In the "point on" sight picture, the tip of the ya cradles the bottom of the center dot. Its alignment may be sensed from the visible length of ya below it. There is a simple test to check that this sense of alignment is not an illusion. Take a long ya, so that the length in front of the yumi at Kai is sufficient to mount a laser pointer. Figure 1 depicts a laser pointer taped to the front end of a 110 *cm* long ya. With the nock oriented as it would be in Kai,

Figure 1: Laser pointer on the front end of a long ya

you want the pointer to hang from the underside of the ya, to mitigate some of the awkwardness in carrying out the Hassetsu from Yugamae to Kai. At Kai, you establish the sight picture with its alignment of the ya, and then look for the laser dot on the target. It helps if this is a crepuscular exercise carried out at dawn or dusk, at a range of about 10 *m*. With the point of the ya cradling the center dot, the laser spot typically appears above it at 12 o'clock. The 12 o'clock orientation indicates that the ya at Kai is in the correct vertical plane which bisects the target. The elevation of the laser dot above the center indicates that ya shot from sufficiently short ranges land above the center dot. This is acceptable since the intention is to measure the right or left drift of ya away from the correct vertical plane at Kai. In an actual shot, the sight picture is acquired and then focused attention to it transitions into peripheral awareness. As the sight picture fluctuates, you "just watch the show," allowing visual proprioception to repeatedly bring the sight picture "back to center." The main attention goes into the body through its expression of Nobiai and Tsumeai.

The shooting test begins with a 14.5 *kg* Rokusun yumi. There are four pairs of test ya, all 102*cm* long and progressively increasing in stiffness. Table 1 lists their stiffnesses relative to the Easton aluminum 2015 shaft. Pairs one and two are aluminum 1913 and 2015. The remaining pairs, bamboo wrapped carbon. The shots are done at a range of 21*m*. Each pair of ya is shot three times, producing six impacts. I found by trial and error that three rounds shot with "strong intention" produces more consistent results

Table 1: Relative stiffnesses of test ya

| ya | stiffness |
|---|---|
| 1 | .75 |
| 2 | 1.00 |
| 3 | 1.22 |
| 5 | 2.29 |

than, say, fifteen rounds more likely subject to somatic and cognitive "drift." Figure 2 is a photograph of the pair of 1913's in the target. The camera is mounted on a tripod whose footprint on the ground is the same for each

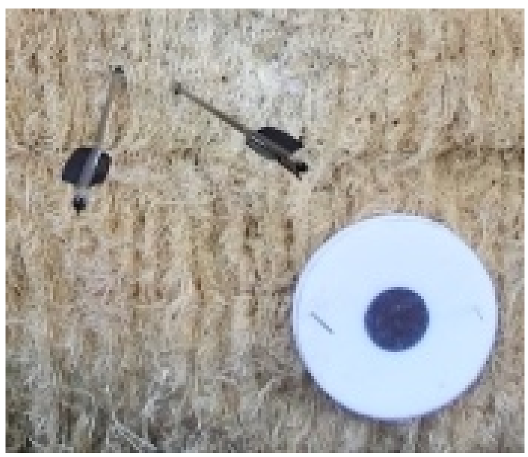

Figure 2: A pair of 1913's in the target

pair of ya. Each photograph is imported into a graphics package called xfig. The center dot is marked by a black disk, and impact points of ya by smaller disks. This done, the photograph is deleted from the figure file. The next photograph is imported and the process repeats. In figure 3, the blue dots mark 1913 impacts, the cyan dots, 2015 impacts, the gold dots, the softer bamboo wrapped carbon and the red dots, the stiffer bamboo wrapped carbon. The horizontal bar sets the scale. It indicates 27 *cm*. A 27 *cm* mato at a range of 21 *m* has the same angular diameter as the standard 36 *cm* mato at 28 *m*.

The group of the softest ya, 1913, drifts to the left by about one mato

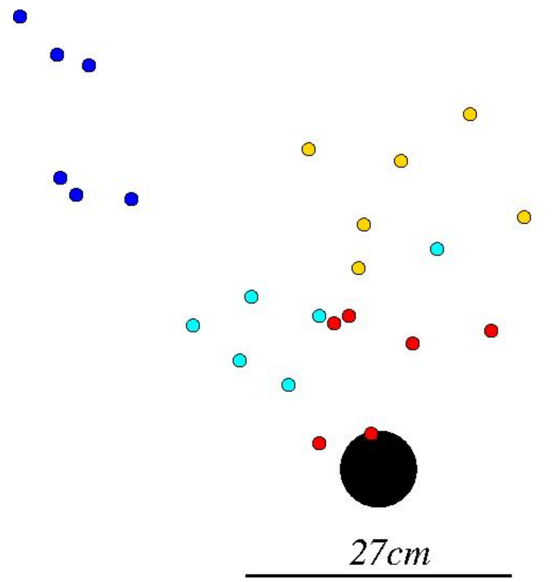


Figure 3: Groupings of ya shot from the range of 21 $m$ with a 14.5 $kg$ Rokusun yumi. Blue dots - 1913 impacts, cyan dots - 2015 impacts, gold dots - the softer bamboo wrapped carbon, red dots - the stiffer bamboo wrapped carbon.

diameter. The 2015 also drift left, by half a mato diameter. The softer bamboo wrapped carbon ya, 22% stiffer than 2015, groups slightly to the right. You might expect that the stiffer bamboo wrapped carbon ya, almost twice as stiff as the softer pair, would group to the right. In fact we see very little drift. Apparently, there is a threshold of stiffness that must be exceeded in order to achieve "correct hitting," so ya flight is confined to the *same* vertical plane defined by its initial alignment in Kai.

Figure 4 shows groupings of the bamboo wrapped carbon ya when they are shot out of a stronger yumi, a 19$kg$ Rokusun. The softer grade of ya now deflects to the left, but the stiffer grade still retains minimal drift. Apparently, the "threshold" stiffness for no drift increases with the strength of the yumi.

The information on ya selection provided by dealers is rather sparse. Sambu-Kyuguten's website gives recommendations for ya shafts based on yumi strength. Table 2 shows appropriate yumi strengths for the very common Easton aluminum shafts. I didn't find recommendations based on ya length as well. The western archer's choice of arrows is informed by *arrow*

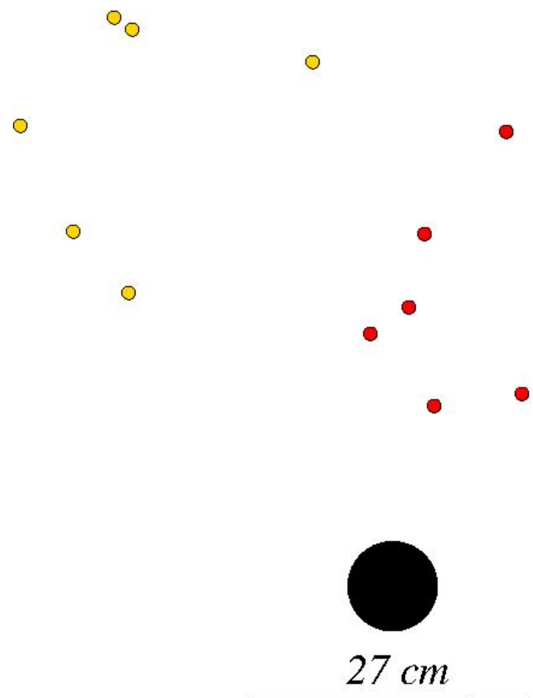


Figure 4: Groups of bamboo wrapped carbon ya shot out of a $19kg$ Rokusun yumi.

*selection charts*. These are arrays, in which each row represents a range of draw force, and the columns, arrow lengths. For instance, the first row of the Easton selection chart corresponds to draw forces between $9.5kg$ and $12.2kg$, the second row, draw forces between $12.2kg$ and $14.5kg$, and so on. The columns correspond to arrow lengths, starting with 23" ($.584m$) for the first column, 24' ($.610m$) for the second column, up to 32" ($.813m$) for the last (10th) column. There is a list of arrow groups. The arrows in each group are close to each other in stiffness. The groups are ordered by increasing stiffness. Each combination of draw force range and arrow length is assigned its appropriate group of arrows.

The arrow lengths covered by the Easton chart are too short for Kyudoka, who require ya lengths between $80cm = .8m$ and $110cm = 1.10m$. A natural *scientific* question is: What is the *physically meaningful* extrapolation of the selection chart to the range of ya lengths?

Table 2: Ranges of yumi strength for Easton aluminum shafts

| Easton aluminum shaft | Yumi strength |
|---|---|
| 1913 | $11kg \rightarrow 13kg$ |
| 2014 | $13kg \rightarrow 15kg$ |
| 2015 | $> 13kg$ |
| 2117 | $> 20kg$ |

**II. Stiffness as a function of draw length and bow strength**

An implicit assumption of traditional selection charts is that recommended shaft stiffness $\mu$ depends mainly on the length $l$ of the arrow and the strength $f$ of the bow. If this is so, we have a simple formula informed by the operational definition of stiffness. It is

$$\mu = cfl^2, \tag{1}$$

where $c$ is a multiplicative constant, the *same* for all entries in the selection chart. Figure 5 depicts a uniform rod with its left endpoint $o$ clamped horizontally and its right end $p$ subject to a vertical load $f$. The load at $p$ exerts a *torque* $\tau$ at the clamped end $o$, which is the length $l$ of $\overline{op}$ times the component $f'$ of the force $f$ perpendicular to $\overline{op}$. That is,

$$\tau = f'l. \tag{2}$$

This torque induces a curvature $\kappa$ of the rod at the clamped end $o$. Recall that the curvature is the inverse radius of the circle which best approximates the bend of the rod near $o$. It is observed that the curvature is directly proportional to the torque which induces it: There is a constant of proportionality $\mu$ so that

$$\tau = \mu\kappa. \tag{3}$$

The constant of proportionality $\mu$ is called the rod's *bending modulus*. We'll use the simpler term *stiffness*. The proportionality (3) between curvature and torque is a special case of the *torque identity* which we encounter in the analysis of the yumi's shape.

In (2) we see that torque is a force times a length. By its definition, curvature is an inverse length. For consistency, the stiffness $\mu$ in (3) is a

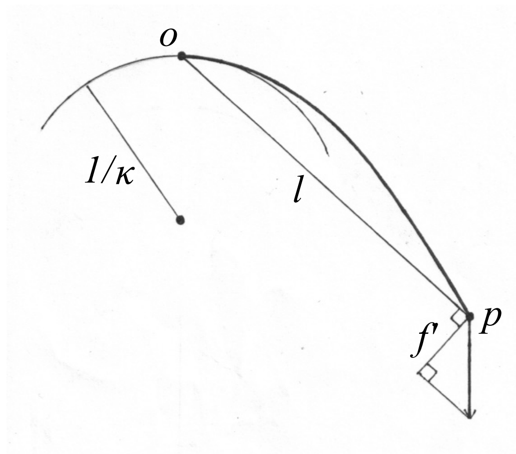


Figure 5: A shaft clamped horizontally at o and subject to a vertical load at the free end p. In practice, the clamping may be achieved by inserting a portion of the shaft snugly into a horizontal hole in a solid wall.

force times length squared. In conventional language, we say: "The units of stiffness are force times length squared." For instance, suppose we measure $l$ in $m$ and $f$ in $kg$. Then $\mu$ is measured in $kg\ m^2$. From this superficial glance at physical units, (1) looks like a no-brainer. Nevertheless, one wonders: Isn't (1) too simple? In practice, one may change the units of length and force, and physical consistency dictates a collateral change in the unit of stiffness. In appendix I, we show how this *scale covariance* singles out the simple formula (1).

A not implausible refinement of (1) says: "It is the *depth* y of the draw (yazuka) that matters, more than the length of the ya." By depth, we mean the actual displacement of the nock (hazu) as we go from the braced, undrawn yumi to the fully drawn yumi at Kai. A rough, common sense approximation says that the depth is the ya length minus the brace height h (Ha). That is,

$$y = l - h. \tag{4}$$

The modification of (1) is

$$\mu = cfy^2. \tag{5}$$

In appendix II, we see how the formula (5) approximates quite well the recommendations of the Easton chart with proper choices of $c$ and $h$,

$$c \approx .1674,\ h \approx 17.2cm \approx 6.73". \tag{6}$$

The brace height of 6.73" which derives from fitting (5) to the Easton chart is reassuring because brace heights $h$ of western bows typically range between 6" and 8" . In applying (5) to Kyudo ya, we retain the value of $c$, but change $h$ to the standard Ha $h \approx 15cm$.

We can discern implications for Kyudoka from (5). Let's say that the strength of a western archer's bow is $f = 17.2kg$, and his arrows are $l = .762m = 30"$ long. The depth of the draw is approximately $y \approx .762m - .172m = .590m$. The Easton chart says that the 2015 aluminum is a good choice of shaft. Now look at a Kyudoka whose yazuka is say, $y' = 90cm$ (hence ya is over 105cm long). Using (5) as the proper relation between yumi strength, ya length and ya stiffness, what is the strength $f'$ of the yumi appropriate for 2015 (according to Easton)? We must have

$$fy^2 = f'y'^2,$$

or

$$f' = f\left(\frac{y}{y'}\right)^2 \approx (17.2kg)\left(\frac{.590}{.90}\right)^2 \approx 7.4kg! \qquad (7)$$

Here is another way to look at it: Let's say that the strength of the yumi is $17.2kg$, same as the western bow. How much stiffer does the ya have to be? Let the new stiffness be $\mu'$. This time, we enforce the equality

$$\frac{\mu}{y^2} = \frac{\mu'}{y'^2},$$

so

$$\frac{\mu'}{\mu} = \left(\frac{y'}{y}\right)^2 \approx 2.32. \qquad (8)$$

More than twice as stiff as 2015! If one accepts the extrapolation of the Easton chart consistent with (5), then "Easton" recommended stiffnesses are *much* greater than what Kyudoka are used to.

To broaden the perspective beyond this simple example, we visualize contours of recommended stiffness in the plane whose axes are arrow length and bow strength. The horizontal axis of figure 6 is arrow length $l$ in $m$, the vertical, bow strength $f$ in $kg$. The bow strength axis is oriented *downwards*, which corresponds to increasing bow strength as you proceed down columns of a selection chart. For any given stiffness $\mu$, (5) defines a curve consisting of all the points $(l, f)$ which yield the given stiffness. As we remarked before, we apply (5) with the standard Ha $h = 15cm$ of yumi. These are the contours of

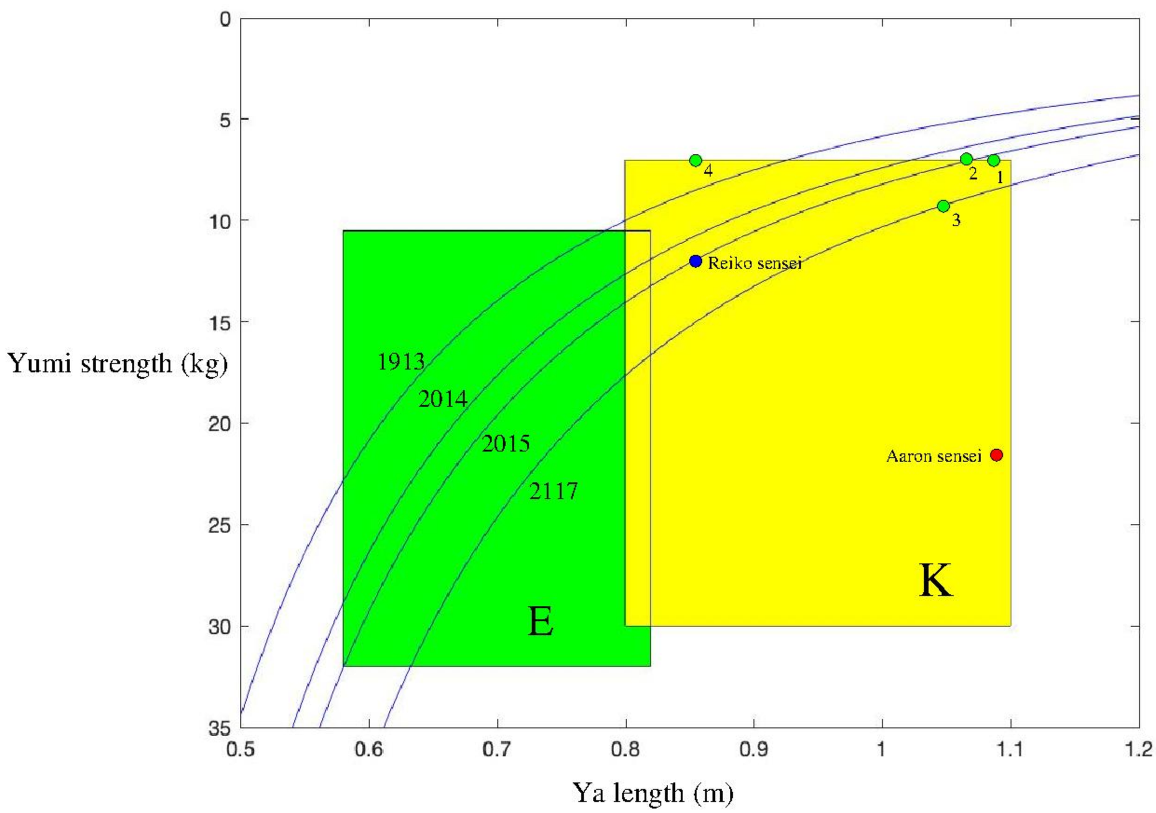


Figure 6: Stiffness contours of Easton aluminum shafts

stiffness in the $l, f$ plane. We've drawn contours corresponding to stiffnesses of the aluminum ya in table 1. For instance, the points $(l, f)$ on the curve marked "2015" represent ya lengths and yumi strengths compatible with 2015 stiffness. The rectangle marked $E$ represents the range of arrow lengths and bow strengths covered by the Easton selection chart. The rectangle marked $K$ represents the range of ya length and yumi strength that "covers" almost all Kyudoka. There is little overlap between $E$ and $K$ because the longest arrows of western archers are barely longer than the shortest ya of Kyudoka. The range of stiffnesses between that of 1913 and 2117 aluminum shafts covers a broad swath of the Easton rectangle $E$, but only the upper left corner of the Kyudo rectangle $K$. The ya length, yumi strength combinations of some Kyudoka are far outside of this region. For instance, the Shinsa ya of

Aaron sensei are $109cm$ long and his yumi are $22kg$ Yonsun. The red dot in figure 6 marks the point $(l, f) = (1.09m, 22kg)$. The ya length and yumi strength combinations of other Kyudoka may be in the region covered by the aluminum ya, but they tend to use ya which are much softer than what is recommended by extrapolation of the Easton chart. For instance, the blue dot $(l, f) = (.85m, 12kg)$ marks the ya length, yumi strength combination of Reiko sensei. According to figure 6, 2015's are appropriate for Reiko sensei, but she actually uses 1913's in daily practice. If we accept the recommendations of ya stiffness according to physics-based extrapolation of the Easton chart, then the commonly available aluminum shafts cover the preferences and needs of western archers, but those of Kyudoka not so much.

**III. Preferences inherited from Kyudo Tradition**

What *are* the preferences and needs of Kyudoka, really? Let's start with preferences informed by long standing Kyudo tradition. What are the characteristics of high-end take-ya produced from single stalks of Yadake bamboo? Four sets of take-ya are examined: Three sets of four take-ya clearly intended for Kyudoka with long yazuka shooting strong yumi, and one set of six, intended for Kyudoka with short yazuka shooting light yumi. The specifications of these ya are presented in table 3. We record the average value of each property. Deviations of diameter and weight from the averages are on the order of a percent, deviations of stiffness, on the order of 5%. Given the measured lengths and stiffnesses of these ya, we can compute the appropriate strengths of yumi from (5). We can locate corresponding points in the $l, f$ plane of figure 6. These green points are labeled by sets. Sets one and two are close to the 2015 contour. In fact, set one ya are 12% stiffer than 2015 and set two, 4% stiffer. These are Shinsa ya of Aaron and Ed sensei respectively, who both use 2015 for routine practice. Set three is close to the contour of

Table 3: Specifications of high end take-ya

| set | length ($cm$) | diameter ($mm$) | weight ($g$) | stiffness ($kg\ m^2$) |
|---|---|---|---|---|
| 1 | 109 | 8.7 | 33.0 | 1.04 |
| 2 | 107 | 8.3 | 30.4 | .97 |
| 3 | 105 | 9.2 | 33.9 | 1.31 |
| 4 | 85 | 7.9 | 23.1 | .58 |

the 2117 shaft. It is probably an outlier: The outer diameter of $9.2mm$ is the largest that can be fitted by yanone in the Sambu Kyuguten inventory. Its larger stiffness, 40% greater than 2015 is probably an upper bound of what is possible for take-ya. Set four is close to the 1913 contour. These Shinsa ya of Reiko sensei are 17% softer than 1913. As noted before, she uses 1913 in routine practice. The stiffnesses of the traditional ya examined here are close to stiffnesses of commonly available aluminum ya.

The weights also match. Knowing the weight of the 2015 shaft per unit

Table 4: Weights of take-ya compared with their aluminum proxies

| | | | | |
|---|---|---|---|---|
| take-ya | 33.0 | 30.4 | 33.9 | 23.1 |
| aluminum | 32.3 | 31.8 | 31.2 | 25.0 |

length, $.25g/cm$, we can estimate the weights of 2015 ya with the same lengths as sets one through three. Similarly, we can estimate the weight of a 1913 ya with the $85cm$ length of set four. Table 4 compares the weights of the take-ya with their aluminum proxies in grams. In summary, the stiffnesses and weights of the commonly available aluminum shafts meet the expectations of Kyudo tradition.

**IV. Ya selection which enables "correct hitting"**

The art of Kyudo requires the flight of the ya to remain in the same vertical plane defined by its initial alignment in Kai. The results of the shooting test in section I suggest that this outcome is enabled by sufficiently stiff ya. The test covers only one ya length, $102cm$, and two yumi strengths, $14.5kg$ and $19kg$. Scale covariance suggests the natural extension to other combinations: We retain the relationship (5) between stiffness, yumi strength and yazuka, but with a readjusted value of the proportionality coefficient $c$. Recall that the bamboo wrapped carbon ya shot from the $14.5kg$ yumi which is 1.22 times as stiff as 2015 hits in line with the mato, but 2015's still deflect to the left. The physical stiffness of the bamboo wrapped carbon ya is $\mu = 1.18kgm^2$. Substituting into (5) this value of stiffness, yazuka $y = 1.02m - .15m = .87m$ and yumi strength $14.5kg$, we find the readjusted coefficient $c \approx .107$, which is roughly *two thirds* of the value $c \approx .1674$ derived from the Easton chart.

Figure 7 shows the readjusted contours of the Easton aluminum ya in the

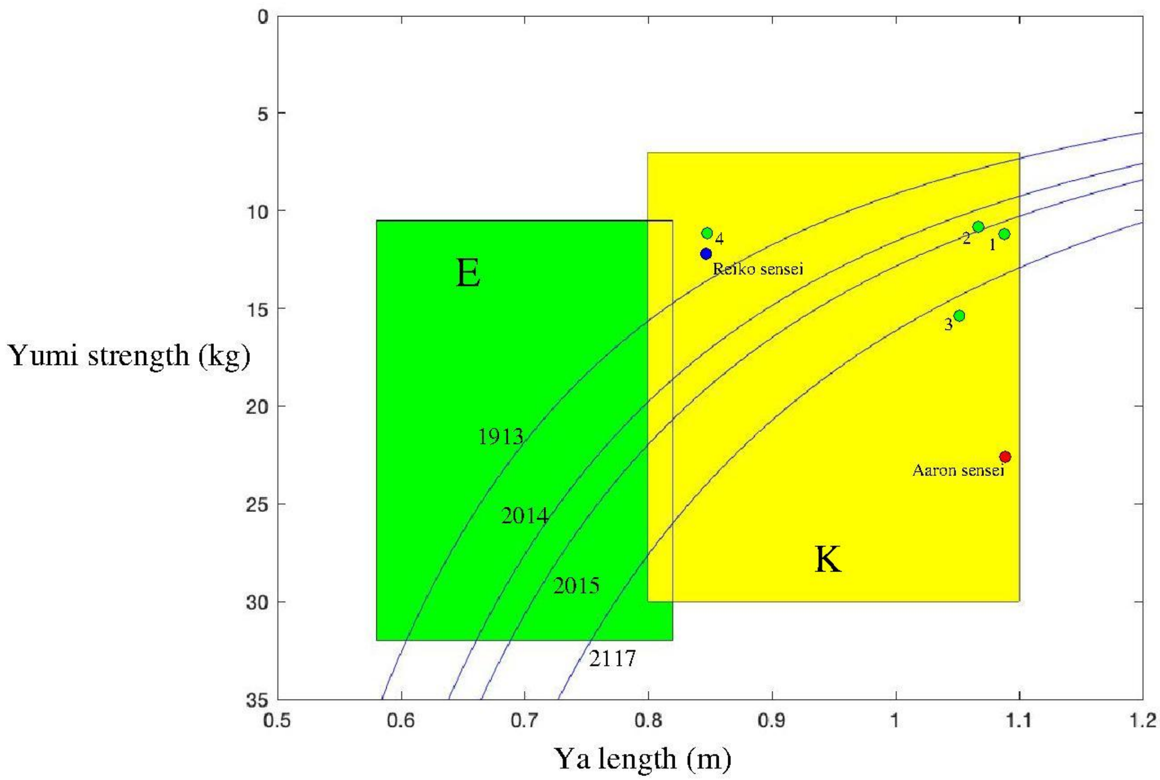


Figure 7: Readjusted stiffness contours

$l, f$ plane. The range of stiffness between that of 1913 and 2117 aluminum shafts now covers a broad swath of the *Kyudo* rectangle. In this new reckoning, the aluminum shafts better accommodate some Kyudoka. In particular, it appears that Reiko sensei's 1913 ya are just a bit stiff for her, but close. For a Kyudoka such as Aaron sensei with a long yazuka shooting a strong yumi, all of the aluminum ya appear to be too soft.

The points in the $l, f$ plane corresponding to the four sets of take-ya have also migrated. Reiko sensei's take-ya appear to be a tad soft for her, but close. The sets 1-3 of take-ya are apparently too soft for Aaron sensei. According to (5) with the readjusted coefficient, his combination of ya length and yumi strength is accommodated by the physical stiffness $\mu \approx (.107)(22kg)(1.09m - .15m)^2 \approx 2.08\ kg\ m^2$. This is 2.24 times as stiff as 2015. The stiffest of the bamboo wrapped carbon ya used in the shooting test is 2.51 times stiffer than 2015, a reasonable match for Aaron sensei.

Finally, we recall the shooting test with the $19kg$ yumi. In that test, the softer bamboo wrapped carbon ya drifted to the left, but the stiffer did not. According to (5) with the readjusted coefficient $c \approx .107$, the threshold stiffness for a 102 $cm$ ya shot from the $19kg$ yumi is $\mu \approx 1.54\ kg\ m^2$. The stiffer bamboo wrapped carbon ya with $\mu \approx 2.51\ kg\ m^2$ should be adequate and so they seem to be.

**V. Critique and conclusions**

The recommendations just presented in the last section sound rather definite. They rest upon the readjusted proportionality coefficient $c \approx .107$, which in turn rests upon the shooting test presented in section I. The shooting test suggests that ya will fly in the correct vertical plane when they exceed a "threshold" stiffness. It is encouraging that the stiffness recommended for Reiko sensei based on the shooting test I performed is close to what she actually uses. Nevertheless, the threshold stiffness may be Kyudoka dependent, and a refined Hanare might permit softer ya. In practice, I'd advise a Kyudoka to not take recommended stiffnesses based on my shooting test as an absolute criterion. Rather, he does his own shooting test to determine his threshold stiffness.

**Appendix I. Scale covariance**

If indeed the recommended shaft stiffness depends only on the draw force $f$ of the bow and the depth of the draw $y$, why *must* the ratio $\frac{\mu}{fy^2}$ be independent of $f$ and $y$? The famous mathematical physicist Hermann Weyl published his book, "Space, soon after after Einstein's geometric theory of gravity appeared. The title is a reminder of a conventional perception, that physical happenings ultimately boil down to movement of matter in space and time. When we say that the length of a ya is $90.5cm$, we mean that it takes 90 and one half steps of a "unit" displacement (a centimeter) to walk from, one end to the other. Similarly, the numbers representing a time duration or a mass are relative to adopted units of time and mass. When we speak of the "units of physical parameters," we acknowledge the relativity of numerical lengths, times and masses to adopted units.

Let's suppose we *change* the adopted units, so numerical values of length, duration and mass are multiplied by *scaling factors* $L, T, M$. When we say "the units of velocity are length divided by time," we mean that the numerical value of velocity changes by the scaling factor $L/T$. Similarly, when we say "the units of acceleration are length divided by time squared," we mean that its value changes by the scaling factor $L/T^2$. Finally, "the units of force are mass times acceleration" means that its numerical value changes by the scaling factor $ML/T^2$. In symbols, we write

$$[f] = F := \frac{ML}{T^2}, \tag{9}$$

where the brackets mean "units of."

The units of an arrow's stiffness are informed by the torque identity (3). Since the torque in (2) is the product of a force times a length, we have

$$[\tau] = FL. \tag{10}$$

Here, $F$ denotes the units of force as in (9). The curvature $\kappa$ as an inverse radius has units

$$[\kappa] = \frac{1}{L}.$$

Hence, balance of units in the torque identity (3) gives

$$FL = [\mu]\frac{1}{L},$$

or

$$[\mu] = FL^2. \tag{11}$$

With $c$ a pure (dimensionless) number, (5) has the correct balance of units, but why is the relationship just (5) and *none other*? Let's suppose that the relationship is

$$\mu = g(f, y) \tag{12}$$

with respect to an original set of length, time and mass units. Suppose we change the these units and $f', y', \mu'$ denote numerical values of draw force, arrow length and stiffness with respect to the new units, so

$$f' = Ff,\ y' = Yy,\ \mu' = FY^2\mu. \tag{13}$$

Denote the relationship between draw force, arrow length and modulus in the new units by

$$\mu' = G(f', y'). \tag{14}$$

From (11), (12), (13) we have

$$FY^2 g(f, y) = G(Ff, Yy), \tag{15}$$

which expresses the consistency between the new selection chart and the old. Specifically, if all yazuka are multiplied by $Y$ and all draw forces by $F$, then physical consistency between the new chart and the old requires us to multiply all stiffness values by $FY^2$. This is an example of *scale covariance*.

The mathematical identity (15) is to hold for the domain of $(f, y)$ covered by the old chart, and arbitrary positive values of $F, Y$. Take any $(f, y)$ in the domain of the old chart, and replace $F$ by $F/f$, and $Y$ by $Y/y$, so (15) becomes

$$\frac{G(F,Y)}{FY^2} = \frac{g(f,y)}{fy^2}. \tag{16}$$

In *any* system of units, the stiffness divided by the draw force and yazuka squared is *one unique constant*.

The conclusions from scale covariance can reach a bit further. It seems reasonable that optimum arrow stiffness may depend on the arrow's mass as well as its length and the draw force of the bow. We can repeat the scale covariance exercise with mass as an additional input parameter. In place of (15), we have

$$FY^2 g(f,y,m) = G(Ff, YyMm),$$

and replacing $F, L, M$ by $F/f, Y/y, M/m$ we arrive at

$$\frac{G(F,Y,M)}{FL^2} = \frac{g(f,y,m)}{fy^2},$$

so in any system of units,

$$\frac{g(f,y,m)}{fy^2} \tag{17}$$

is one unique constant, and there is no dependence on mass.

**Appendix II. Summary of the Easton selection chart**

We show how the simple formula (5) for stiffness approximates the recommendations of the Easton chart. First, we present its content seen through the lens of physics. Recall that for each range of draw force and arrow length, there is a group of arrows with "appropriate" stiffnesses. To discern subtle trends, we summarize each group by the geometric mean of bending moduli. We use the geometric mean because we examine the selection chart through the lens of scale covariance. A further simplification is to collapse each range of draw forces into a single force value, the geometric mean of endpoint values. For instance, the range of draw force from $20.0kg$ to $21.8kg$ is represented by the single value $\sqrt{(20.0)(21.8)}kg \approx 20.9kg$.

The selection chart as presented to the archery public does not mention bending modulii. The traditional archery engineer's measure of stiffness is the so-called *spine*: The particular kind of shaft being tested is prepared with a 29" length, and it is supported at two points 28" apart. A $1.94lb$ weight is hung from the midpoint, and the arrow's *spine* is the deflection in inches. This is how the archery public understands stiffness: Smaller spine, greater stiffness. A simple elastostatic analysis yields a conversion of the spine $s$ into bending modulus $\mu$ in the units of $kgm^2$. (Here, $kg$ is understood as a unit of force.) The formula is

$$\mu \approx \frac{.518}{s}\ kg\ m^2. \tag{18}$$

Let's work an example of how the numbers are crunched. In the Easton chart, the group of arrows assigned to the draw force range between $20.0kg$ and $21.8kg$ and arrow length of 29" $= .737m$ has 14 members. The list of spines is

$$.470, .450, .470, .480, .500, .440, .490, .500, .500, .500, .505, .460, .510, .475.$$

We convert these spines into modulii by (18). The geometric mean modulus, $1.075kg\ m^2$, is obtained by multiplying these 14 modulii together and taking the 1/14th power. We assign geometric mean modulus $1.075kgm^2$ to this group of arrows. The average percentage variation from the geometric mean is 3.9%.

Table 5: List of stiffnesses

| group | stiffness $kg\ m^2$ | group | stiffness $kg\ m^2$ |
|---|---|---|---|
| 1 | .263 | 10 | .918 |
| 2 | .334 | 11 | .982 |
| 3 | .383 | 12 | 1.075 |
| 4 | .436 | 13 | 1.210 |
| 5 | .533 | 14 | 1.302 |
| 6 | .609 | 15 | 1.419 |
| 7 | .679 | 16 | 1.619 |
| 8 | .761 | 17 | 1.715 |
| 9 | .817 | 18 | 1.955 |

Table 6: Recommended groups

| force | $.584m$ | $.610m$ | $.635m$ | $.660m$ | $.686m$ | $.711m$ | $.737m$ | $.762m$ | $.787m$ | $.813m$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $10.8kg$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | - | - | - |
| $13.3kg$ | 2 | 3 | 4 | 5 | $6^*$ | 7 | 8 | 9 | - | - |
| $15.4kg$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | - |
| $17.2kg$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| $19.0kg$ | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| $20.9kg$ | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $22.7kg$ | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| $24.9kg$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| $27.2kg$ | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 17 |
| $29.3kg$ | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 17 | 18 |
| $31.9kg$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 17 | 18 | - |

Table 5 lists the stiffnesses associated with 18 arrow groups, in order of increasing stiffness. Table 6 presents the assignments of groups to the various draw force, arrow length combinations. Some combinations have *no* assigned group, because there are no arrows appropriate for them.

A peculiar feature of table 6 jumps out: Along any "diagonal" generated by matching incremental decreases of draw force with corresponding incremental increases of arrow length, the assigned group of arrows remains the *same*. The increments of arrow length are all the same, $\Delta l = 1" = .0254m$, and the increments of draw force are nearly constant, near $\Delta f = 2.11kg$. So according to the Easton chart, stiffness contours in the arrow length, draw force plane are *parallel straight lines*. In figure 6, we see the same green and yellow rectangles that cover the ranges of arrow length and draw force for Western archers and Kyudoka. The line segment in the green rectangle labeled "10" represents the stiffness contour of an arrow in group 10 (slightly softer than 2015) according to the Easton chart. The dashed line segment represents the extrapolation to *zero* draw force, and an arrow length of $.945m$. This is absurd. An arrow does not need any stiffness at all if it is not shot! Apparently, the parallel line stiffness contours of the Easton chart represent an *approximation* confined to the ranges of arrow lengths and draw forces that it covers (the green rectangle). Bear in mind that the Easton chart has been a practical guide to shaft selection for decades, so the approximation must work in real life. For Western archers: The curve in figure 8 is the stiffness contour for a group 10 shaft according to the simple formula (3). Inside the green rectangle, it is not far from the line contour. For Kyudoka whose ya length, yumi strength combinations are in the yellow rectangle, the extrapolation of the Easton line is questionable.

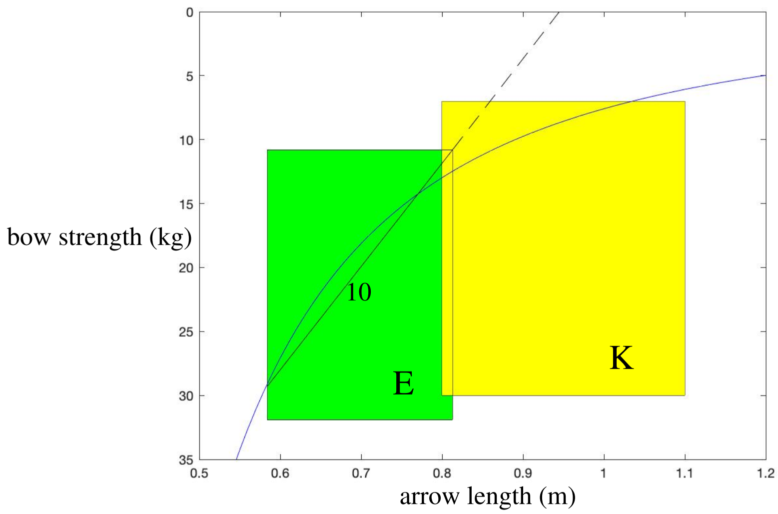


Figure 8: Extrapolations of stiffness contours

Our approach is to take the simple formula (5) as more fundamental because it embodies scale covariance: Once you decide that stiffness depends primarily on draw force and depth of draw, the formula (5) is inevitable, as shown in appendix I. The question remains: How do you choose the dimensionless constant $c$ and the effective brace height $h$ so (5) best approximates the actual Easton chart over *its* range of arrow lengths and draw force.

Suppose that the Easton chart followed (5) *exactly* for some values of $c$ and $h$. For each combination of arrow length $l$ and draw force $f$ in the chart, we would have the *same* value of

$$c = \frac{\mu}{f(l-h)^2}. \tag{19}$$

But since it doesn't we get some collection of different values as we range over all the combinations of $l$ and $f$. For a fixed value of the brace height $h$,

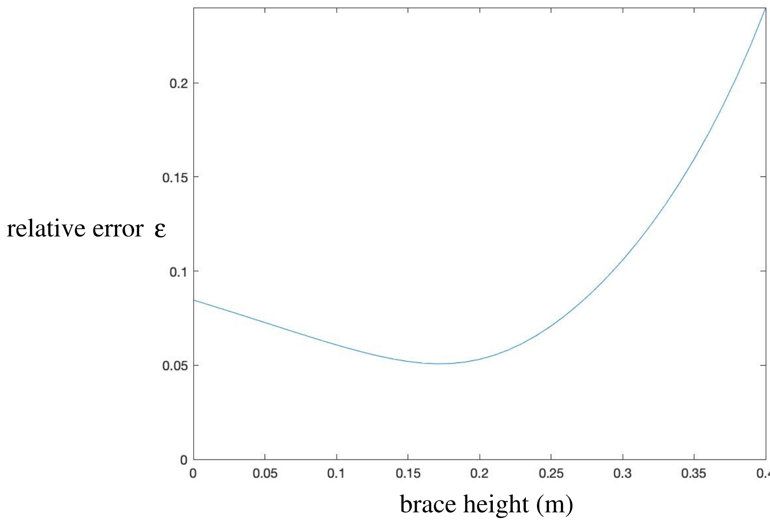


Figure 9: Relative error versus brace height

compute the *mean* $\langle c\rangle$ of these values, and the *root mean square deviation*

$$\sqrt{\langle c-\langle c\rangle\rangle^2}=\sqrt{\langle c^2\rangle-\langle c\rangle^2}. \tag{20}$$

The plan is to *choose* the effective brace height $h$ which minimizes the *relative error*

$$\epsilon=\frac{\sqrt{\langle c^2\rangle-\langle c\rangle^2}}{\langle c\rangle}. \tag{21}$$

Figure 9 is the graph of this relative error over the range $0<h<.4m$. There is a clear minimum value of $\epsilon\approx .052$ at $h\approx .172m=17.2cm$. For this effective brace height, the range of variations in values of $c$ from the mean value $\langle c\rangle\approx .1674$ is about 5%. This is the basis for the values $c\approx .1674,\ \ h\approx 17.2cm$ in (5).